

Delta(1232) contribution to real photon radiative corrections for elastic electron-proton scattering

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Abstract. Here we consider a contribution of Delta(1232) resonance to real photon radiative corrections for elastic ep -scattering. The effect is found to be small for past experiments to study unpolarized cross section as well as for the recent VEPP-3 experiment to investigate two-photon exchange effects by precision measurement of $e^\pm p$ -scattering cross sections ratio.

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1. Introduction

The electromagnetic form factors of the proton ($G_{E,M}$) contain information about its internal structure. The Rosenbluth separation method [1] have been used since 1950s to extract the form factors from the unpolarized electron-proton scattering cross section [2, 3, 4, 5, 6, 7, 8, 9]. It became possible to extract the ratio G_E/G_M with the polarization transfer method [10] since 2000s, and the results obtained by the two methods unexpectedly contradict each other [11, 12, 13, 14, 15, 16].

It is suggested that a more accurate account of the two-photon exchange (TPE) effects in the experiments with unpolarized particles can reduce the discrepancy [17]. Theoretical investigations of TPE effects have been performed using various models (see reviews [18, 19] and references therein). For example, in the hadronic model the TPE amplitude can be approximated by successive consideration of the virtual proton, $\Delta(1232)$ and higher resonances in the proton intermediate state [20, 21, 22, 23].

From the experimental point of view the TPE effects can be studied in comparison of elastic electron-proton and positron-proton scattering cross sections. There are three new experiments [24, 25, 26] aiming to precise measure the cross section ratio $R = \sigma(e^+p)/\sigma(e^-p)$. In the leading order of the electromagnetic coupling constant we have

$$R = 1 - 2\delta_{2\gamma} - 2\delta_{\text{brem,odd}}, \quad (1)$$

where the virtual radiative correction $\delta_{2\gamma}$ comes from the interference of the TPE amplitude with the one-photon exchange (Born) amplitude; the C-odd real radiative correction $\delta_{\text{brem,odd}}$ originates from the interference of the electron and proton bremsstrahlung amplitudes. The both corrections have infrared divergences which cancel in their sum. The Mo-Tsai convention [27] is commonly used to regularize and pick out the soft photon terms:

$$\delta_{2\gamma} = \delta_{2\gamma}^{\text{soft}} + \delta_{2\gamma}^{\text{hard}}, \quad \delta_{\text{brem,odd}} = \delta_{\text{brem,odd}}^{\text{soft}} + \delta_{\text{brem,odd}}^{\text{hard}}. \quad (2)$$

To extract the TPE effects contribution $\delta_{2\gamma}^{\text{hard}}$ from R it is necessary to exclude the “hard” part of real radiative corrections $\delta_{\text{brem,odd}}^{\text{hard}}$. This correction strongly depends on particular experimental conditions. In this paper we will primarily address to the experiment at the VEPP-3 storage ring [26]. The ESEPP event generator [28] was used to calculate $\delta_{\text{brem,odd}}^{\text{hard}}$ for the VEPP-3 experiment. It takes into account virtual proton intermediate state in proton bremsstrahlung. Using the hadronic model one has to consider resonances in the intermediate state. Their contributions do not have infrared divergences since the spectrum of bremsstrahlung photons in this case is different from the infrared $d\omega/\omega$, because the resonances have masses distinct from the mass of the proton, that prevents the appearance of ω in the denominator. We can expect that $\Delta(1232)$ will give the leading contribution since it is the lowest resonance as well as it has considerable branching for the decay $\Delta \rightarrow p\gamma$. As we will show in the following a naive estimate gives significant contribution to the radiative corrections, and only a more accurate calculation ensures us that this correction is actually small.

2. Transition vertexes and form factors

Let us consider the process $\gamma(q) p(p) \rightarrow \Delta(p_\Delta)$. We will use the following prescription for the transition matrix element:

$$i\mathcal{M}_{\gamma p \rightarrow \Delta} = iZe J_{p \rightarrow \Delta}^\nu(p, p_\Delta) \epsilon_\nu(q), \quad (3)$$

where the transition current

$$J_{p \rightarrow \Delta}^\nu(p, p_\Delta) = \bar{U}_\beta(p_\Delta) \Gamma_{\gamma p \rightarrow \Delta}^{\nu\beta}(p_\Delta, q) U(p), \quad (4)$$

the electron charge $e = -|e|$; $Z = +1$ is preserved as a matter of traditional notation for radiative corrections to distinguish C-odd and C-even terms; $\epsilon_\nu(q)$ is the photon polarization vector, $U(p)$ is the proton bispinor, Δ is described with the help of the spin-3/2 wave function $U_\beta(p)$.

The electromagnetic current is hermitian. From (4) one can derive the following relation between the transition vertexes in the direct ($\gamma p \rightarrow \Delta$) and inversed ($\Delta \rightarrow \gamma p$) processes as it was emphasized in [23]:

$$\Gamma_{\Delta \rightarrow \gamma p}^{\nu\beta}(p_\Delta, q) = \gamma^0 \left(\Gamma_{\gamma p \rightarrow \Delta}^{\nu\beta}(p_\Delta, q) \right)^\dagger \gamma^0, \quad (5)$$

where in both sides p_Δ stands for the Δ momentum, q is the photon momentum, and $p = p_\Delta - q$ is the proton momentum.

Zhou and Yang [23] make use of the following parameterization:

$$\begin{aligned} \Gamma_{\gamma p \rightarrow \Delta}^{(ZY), \nu\beta}(p_\Delta, q) = & -\sqrt{\frac{2}{3}} \frac{1}{2M_\Delta^2} \gamma^5 \left\{ G_1(q^2) [g^{\nu\beta} \hat{q} \hat{p}_\Delta - p_\Delta^\nu \hat{q} \gamma^\beta - \gamma^\beta \gamma^\nu (p_\Delta \cdot q) + \hat{p}_\Delta \gamma^\nu q^\beta] \right. \\ & + G_2(q^2) [p_\Delta^\nu q^\beta - g^{\nu\beta} (p_\Delta \cdot q)] \\ & \left. - \frac{G_3(q^2)}{M_\Delta} [q^2 (p_\Delta^\nu \gamma^\beta - g^{\nu\beta} \hat{p}_\Delta) + q^\nu (q^\beta \hat{p}_\Delta - \gamma^\beta (p_\Delta \cdot q))] \right\}, \end{aligned} \quad (6)$$

where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and M_Δ is the $\Delta(1232)$ mass. The form factors G_i depend only on q^2 , so they are real functions in the region $q^2 < 0$, where we do not have any discontinuities. In the following to provide numerical results we apply the model from [23], which defines the form factors

$$G_i(q^2) = g_i F_\Delta^{(1)}(q^2), \quad i = 1, 2, 3, \quad (7)$$

by the set of parameters $\{g_1, g_2, g_3\} = \{6.59, 9.08, 7.12\}$ (the values of the form factors at $q^2 = 0$) and the q^2 -dependant factors

$$\begin{aligned} F_\Delta^{(1)}(q^2) = F_\Delta^{(2)}(q^2) &= \left(\frac{-\Lambda_1^2}{q^2 - \Lambda_1^2} \right)^2 \frac{-\Lambda_3^2}{q^2 - \Lambda_3^2}, \\ F_\Delta^{(3)}(q^2) &= \left(\frac{-\Lambda_1^2}{q^2 - \Lambda_1^2} \right)^2 \frac{-\Lambda_3^2}{q^2 - \Lambda_3^2} \left[a \frac{-\Lambda_2^2}{q^2 - \Lambda_2^2} + (1-a) \frac{-\Lambda_4^2}{q^2 - \Lambda_4^2} \right], \end{aligned} \quad (8)$$

with $\Lambda_1 = 0.84$ GeV, $\Lambda_2 = 2$ GeV, $\Lambda_3 = \sqrt{2}$ GeV, $\Lambda_4 = 0.2$ GeV, $a = -0.3$.

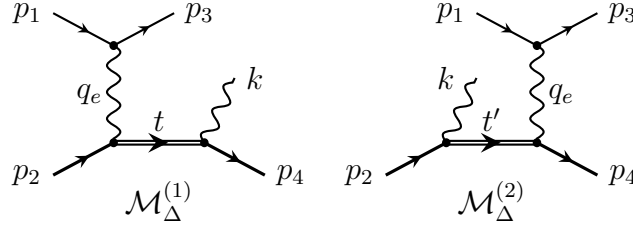


Figure 1. Feynman diagrams for the proton bremsstrahlung with Δ in the intermediate state

There is a more commonly used parametrization by Jones and Scadron [29] in terms of the magnetic $G_M^*(q^2)$, electric $G_E^*(q^2)$ and Coulomb $G_C^*(q^2)$ form factors:

$$\begin{aligned} \Gamma_{\gamma p \rightarrow \Delta}^{(JS), \nu\beta}(p_\Delta, q) = & -i\sqrt{\frac{2}{3}} \frac{3(M_\Delta + M_p)}{2M_p [(M_\Delta + M_p)^2 - q^2]} \left\{ G_M^*(q^2) \epsilon^{\nu\beta\rho\sigma}(p_\Delta)_\rho q_\sigma \right. \\ & + G_E^*(q^2) \left[\frac{4 \epsilon^{\nu\tau\rho\sigma}(p_\Delta)_\rho q_\sigma g_{\tau\tau'} \epsilon^{\beta\tau'\lambda\kappa}(p_\Delta)_\lambda q_\kappa}{(M_\Delta - M_p)^2 - q^2} (i\gamma^5) - \epsilon^{\nu\beta\rho\sigma}(p_\Delta)_\rho q_\sigma \right] \\ & \left. + G_C^*(q^2) \frac{2(q^2 p_\Delta^\nu - (q \cdot p_\Delta) q^\nu) q^\beta}{(M_\Delta - M_p)^2 - q^2} (i\gamma^5) \right\}, \end{aligned} \quad (9)$$

where $\epsilon^{0123} = +1$, and M_p is the proton mass.

Considering the matrix element $\mathcal{M}_{\gamma p \rightarrow \Delta}$ for the definite helicities of the particles we can find the relations between the two set of form factors:

$$\begin{aligned} G_M^*(q^2) = & \frac{M_p}{3(M_\Delta + M_p)} \left[\frac{(M_\Delta + M_p)^2 - q^2}{M_\Delta^2} G_1(q^2) \right. \\ & \left. - \frac{M_\Delta^2 - M_p^2 + q^2}{2M_\Delta^2} (G_1(q^2) - G_2(q^2)) - \frac{-q^2}{M_\Delta^2} G_3(q^2) \right], \\ G_E^*(q^2) = & \frac{M_p}{3(M_\Delta + M_p)} \left[-\frac{M_\Delta^2 - M_p^2 + q^2}{2M_\Delta^2} (G_1(q^2) - G_2(q^2)) - \frac{-q^2}{M_\Delta^2} G_3(q^2) \right], \\ G_C^*(q^2) = & \frac{2M_p}{3(M_\Delta + M_p)} \left[-(G_1(q^2) - G_2(q^2)) + \frac{(M_\Delta^2 - M_p^2 + q^2)}{2M_\Delta^2} G_3(q^2) \right]. \end{aligned} \quad (10)$$

These formulas for $q^2 = 0$ can be found in [23]. To check them for $q^2 \neq 0$ it is possible to combine expressions from [23] and from the review [30].

3. A rough estimate for Delta(1232) contribution to real radiative correction

Using the hadronic model we have to consider two Feynman diagrams presented on the Figure 1. To find relevant contribution to radiative corrections we must calculate the square of absolute value of their sum and the interference of these amplitudes with the amplitudes of electron and proton bremsstrahlung. Then we have to integrate the result over the final particles phase space taking into account the particular experimental conditions. Divided by the elastic process cross section it yields the contribution δ_Δ to real radiative corrections for electron-proton scattering. We will implement this

procedure in the next section. But one can note that the first amplitude on the Figure 1 has “resonant” behavior: the virtual photon energy transfer makes the intermediate Δ to be closer to the real particle pole position so the square of this amplitude might be dominant and give a reasonable approximation. It is worth to note that both amplitudes are gauge invariant separately due to the interaction vertex structure, that is the reason why they can be treated independently. Taking this into account a very rough estimate of δ_Δ can be obtained if we consider the bremsstrahlung as two successive processes $ep \rightarrow e\Delta$ and $\Delta \rightarrow p\gamma$ and assume that all photons from the decay contribute to real radiative corrections:

$$\delta_\Delta \simeq \frac{d\sigma'/d\Omega}{d\sigma/d\Omega} \frac{\Gamma_{\Delta \rightarrow p\gamma}}{\Gamma_\Delta}, \quad (11)$$

where $d\sigma/d\Omega$ is the differential cross section for elastic process $ep \rightarrow ep$ with respect to the electron scattering angle $d\Omega$; $d\sigma'/d\Omega$ is the differential cross section for the process $ep \rightarrow e\Delta$ with the same electron scattering angle; $\Gamma_{\Delta \rightarrow p\gamma}$ and Γ_Δ are partial and full widths of $\Delta(1232)$, their ratio defines probability for the decay to be electromagnetic.

To use the estimate (11) we need the cross section ratio for the elastic scattering $e(p_1)p(p_2) \rightarrow e(p_3)p(p_4)$ and the process $e(p_1)p(p_2) \rightarrow e(p'_3)\Delta(p'_4)$. In this section the quantities without primes refer to $ep \rightarrow ep$, and the quantities with primes to $ep \rightarrow e\Delta$. The initial state is the same for the both processes: the electron and proton 4-momenta are $p_1 = \{\varepsilon_1, \mathbf{p}_1\}$ and $p_2 = \{M_p, 0\}$ correspondingly. The final states are different: electron and proton(Δ) 4-momenta are $p_3^{(\prime)} = \{\varepsilon_3^{(\prime)}, \mathbf{p}_3^{(\prime)}\}$ and $p_4^{(\prime)} = \{\varepsilon_4^{(\prime)}, \mathbf{p}_4^{(\prime)}\}$. The momentum transfers are $q^{(\prime)} = p_1 - p_3^{(\prime)} = p_4^{(\prime)} - p_2$. In addition to the proton M_p , and the $\Delta(1232)$ mass M_Δ we will use the electron mass m (in most cases we consider ultrarelativistic electrons $\varepsilon_1, \varepsilon_3^{(\prime)} \gg m$ and $(q^{(\prime)})^2 \gg m^2$).

The matrix elements have the following form:

$$i\mathcal{M} = -\frac{iZe^2}{q^2} j_\nu(p_1, p_3) J_p^\nu(p_2, p_4), \quad (12)$$

$$i\mathcal{M}' = -\frac{iZe^2}{q'^2} j_\nu(p_1, p'_3) J_{p \rightarrow \Delta}^\nu(p_2, p'_4), \quad (13)$$

where the electron current

$$j^\nu(p_1, p_3) = \bar{u}(p_3) \gamma^\nu u(p_1), \quad (14)$$

the proton current

$$J_p^\nu(p_2, p_4) = \bar{U}(p_4) \Gamma_{\gamma p \rightarrow p}^\nu(q) U(p_2), \quad (15)$$

and the transition current $J_{p \rightarrow \Delta}$ is defined by (4).

The proton electromagnetic vertex is parametrized with the help of two form factors $F_{1,2}(q^2)$:

$$\Gamma_{\gamma p \rightarrow p}^\nu(q) = F_1(q^2) \gamma^\nu - F_2(q^2) \frac{[\gamma^\nu, \hat{q}]}{4M_p}. \quad (16)$$

The proton electric G_E and magnetic G_M form factors can be expressed in terms of $F_{1,2}$ as follows:

$$G_M(q^2) = F_1(q^2) + F_2(q^2), \quad G_E(q^2) = F_1(q^2) + \frac{q^2}{4M_p^2} F_2(q^2). \quad (17)$$

The differential cross sections for unpolarized particles in the case of ultrarelativistic electrons for the same electron scattering angle θ (azimuthal symmetry leads to $d\Omega = 2\pi d\cos\theta$) are

$$\frac{d\sigma^{(\prime)}}{d\Omega} = \frac{1}{(4\pi)^2} \frac{1}{4M_p^2\eta} \frac{\varepsilon_3^{(\prime)}}{\varepsilon_1} \sum^- |\mathcal{M}^{(\prime)}|^2, \quad (18)$$

where

$$\eta = 1 + \frac{2\varepsilon_1}{M_p} \sin^2 \frac{\theta}{2}, \quad \frac{\varepsilon_3}{\varepsilon_1} = \frac{1}{\eta}, \quad \frac{\varepsilon_3'}{\varepsilon_1} = \frac{1}{\eta} \left(1 - \frac{M_\Delta^2 - M_p^2}{2M_p\varepsilon_1} \right). \quad (19)$$

The squared matrix elements can be presented as the products of current tensors:

$$\sum^- |\mathcal{M}|^2 = \frac{Z^2 e^4}{(q^2)^2} L_{\nu\rho}(p_1, p_3) T_p^{\nu\rho}(p_2, p_4) \quad (20)$$

and

$$\sum^- |\mathcal{M}'|^2 = \frac{Z^2 e^4}{(q'^2)^2} L_{\nu\rho}(p_1, p'_3) T_{p \rightarrow \Delta}^{\nu\rho}(p_2, p'_4). \quad (21)$$

The electron current tensor is

$$L^{\nu\rho}(p_1, p_3) = \sum^- j^\nu(p_1, p_3) j^{\dagger\rho}(p_1, p_3). \quad (22)$$

Proton T_p and transition $T_{p \rightarrow \Delta}$ current tensors have the same form. All of them are either well known or could be calculated straightforward. We present appropriate formulas in the Appendix A.

The convolution in (20) leads to the well known Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 \alpha^2 \cos^2 \frac{\theta}{2}}{4\varepsilon_1^2 \eta \sin^4 \frac{\theta}{2}} \frac{\tau G_M^2(q^2) + \epsilon G_E^2(q^2)}{\epsilon(1 + \tau)}, \quad (23)$$

where $\alpha = e^2/4\pi$,

$$\tau = \frac{-q^2}{4M_p^2}, \quad \epsilon = \left(1 + 2(1 + \tau) \tan^2 \frac{\theta}{2} \right)^{-1}. \quad (24)$$

Using (21) for the process $ep \rightarrow e\Delta$ one can find

$$\begin{aligned} \frac{d\sigma'}{d\Omega} &= \frac{Z^2 \alpha^2 \cos^2 \frac{\theta}{2} (M_p + M_\Delta)^2}{4\varepsilon_1^2 \eta \sin^4 \frac{\theta}{2} 4M_p^2} \\ &\times \frac{\tau' \left(G_M^{*2}(q'^2) + 3G_E^{*2}(q'^2) + \epsilon' \frac{-q'^2}{M_\Delta^2} G_C^{*2}(q'^2) \right)}{\epsilon'(1 + \tau')}, \end{aligned} \quad (25)$$

where

$$\tau' = \frac{-q'^2}{(M_\Delta + M_p)^2}, \quad \epsilon' = \left(1 + 2 \left(1 + \frac{\nu^2}{-q'^2} \right) \tan^2 \frac{\theta}{2} \right)^{-1}, \quad (26)$$

with $\nu = \varepsilon_1 - \varepsilon'_3 = (M_\Delta^2 - M_p^2 - q'^2)/(2M_p)$.

If we consider the cross sections ratio

$$\frac{d\sigma'/d\Omega}{d\sigma/d\Omega} = \frac{\epsilon(1+\tau)}{\epsilon'(1+\tau')} \frac{(M_p + M_\Delta)^2}{4M_p^2} \times \frac{\tau' \left(G_M^{*2}(q'^2) + 3G_E^{*2}(q'^2) + \epsilon' \frac{-q'^2}{M_\Delta^2} G_C^{*2}(q'^2) \right)}{\tau G_M^2(q^2) + \epsilon G_E^2(q^2)}, \quad (27)$$

and take into account the conditions of the VEPP-3 experiment [26] we will find that the cross section ratio is about 1. So for the estimate (11) of Δ contribution to real radiative corrections for elastic ep -scattering we could write the following expression

$$\delta_\Delta \simeq \frac{d\sigma'/d\Omega}{d\sigma/d\Omega} \frac{\Gamma_{\Delta \rightarrow p\gamma}}{\Gamma_\Delta} \simeq 0.5\%. \quad (28)$$

We used the branching $\text{Br}(\Delta \rightarrow N\gamma) = 0.55 - 0.65\%$ from PDG [31] and the values of the transition form factors $G_{M,E,C}^*$ derived from the parametrization (6) and the equations (10). It is a very rough estimate, moreover the numerical value seems significant for the VEPP-3 experimental results where the TPE effect is of order 1%. So in the following we will present a more accurate calculation.

4. Proton bremsstrahlung with Delta(1232) in the intermediate state

Hereafter we consider the process $e(p_1)p(p_2) \rightarrow e(p_3)p(p_4)\gamma(k)$, which contributes to real photon radiative corrections. There are two Feynman diagrams for the proton bremsstrahlung with Δ in the intermediate state (see Figure 1):

$$i\mathcal{M}_\Delta = i\mathcal{M}_\Delta^{(1)} + i\mathcal{M}_\Delta^{(2)}, \quad (29)$$

where

$$i\mathcal{M}_\Delta^{(1)} = \frac{iZ^2 e^3}{q_e^2} j_\nu(p_1, p_3) \epsilon_\mu^*(k) \frac{\bar{U}(p_4) \Delta^{\mu\nu}(t; k, q_e) U(p_2)}{t^2 - M_\Delta^2 + i\Gamma_\Delta M_\Delta}, \quad (30)$$

with

$$\Delta^{\mu\nu}(t; k, q_e) = \Gamma_{\Delta \rightarrow \gamma p}^{\mu\alpha}(t, k) (\hat{t} + M_\Delta) \mathcal{P}_{\alpha\beta}(t) \Gamma_{\gamma p \rightarrow \Delta}^{\nu\beta}(t, q_e), \quad (31)$$

and

$$i\mathcal{M}_\Delta^{(2)} = \frac{iZ^2 e^3}{q_e^2} j_\nu(p_1, p_3) \epsilon_\mu^*(k) \frac{\bar{U}(p_4) \Delta^{\nu\mu}(t'; -q_e, -k) U(p_2)}{t'^2 - M_\Delta^2} \quad (32)$$

where we use $t = p_1 + p_2 - p_3$ and $t' = p_2 - k$, the electron momentum transfer is $q_e = p_1 - p_3$, and the Δ propagator contains [23]

$$\mathcal{P}^{\alpha\beta}(t) = -g^{\alpha\beta} + \frac{\gamma^\alpha \gamma^\beta}{3} + \frac{\hat{t} \gamma^\alpha t^\beta + t^\alpha \gamma^\beta \hat{t}}{3t^2}. \quad (33)$$

We save the width Γ_Δ for the first term $\mathcal{M}_\Delta^{(1)}$ because there is the resonance region when t^2 is close to M_Δ^2 , and this region can give the main contribution of Δ to real radiative corrections. In the second term $\mathcal{M}_\Delta^{(2)}$ the real photon emission moves away

the amplitude from the resonance, so the omitted width can not sufficiently change the results.

We will use additional simplification leaving in results only the terms, which have minimal powers of the photon energy ω and the difference $M_\Delta - M_p$ assuming

$$\omega \ll M_p, \quad M_\Delta - M_p \ll M_p. \quad (34)$$

The first limit is a part of the traditional soft photon approximation. This approximation is more suitable to experiments with magnetic spectrometers to study the electron-proton elastic scattering cross section, where energy restrictions on the unobservable photon and proton are rather strict. In the VEPP-3 experiment energy cuts are conservative, so the contribution of hard photons can be sufficient. The second limit allows us to sufficiently simplify the result of traces calculation in $|\mathcal{M}_\Delta|^2$ and in the interference of \mathcal{M}_Δ with the amplitude of electron bremsstrahlung. We do not modify the denominators of Δ propagator since they define the resonance behavior of the amplitude \mathcal{M}_Δ . As for the numerators in the soft photon limit this approximation means expansion in terms of the small ratio $(M_\Delta - M_p)/M_p$ and saving only the leading terms.

4.1. Delta(1232) contribution to elastic cross section measurement experiments

The square of the matrix element $|\mathcal{M}_\Delta|^2$ leads to C -even contribution, so it has no influence on the ratio R of the elastic $e^\pm p$ -scattering cross sections in the leading order of electromagnetic coupling constant. But, in principal, it could affect the results of the experiments to measure the unpolarized $e^- p$ -scattering cross section.

As we supposed above the leading contribution of $|\mathcal{M}_\Delta|^2$ to bremsstrahlung differential cross section comes from

$$\sum \left| \mathcal{M}_\Delta^{(1)} \right|^2 = \frac{Z^4 e^6}{(q_e^2)^2} \frac{L_{\nu\nu'}(p_1, p_3) H^{\nu\nu'}(t; k, q_e)}{(t^2 - M_\Delta^2)^2 + \Gamma_\Delta^2 M_\Delta^2}, \quad (35)$$

where

$$H^{\nu\nu'}(t; k, q_e) = \frac{(-g_{\mu\mu'})}{2} \text{Tr} \left[(\hat{p}_4 + M_p) \Delta^{\mu\nu}(t; k, q_e) (\hat{p}_2 + M_p) \gamma^0 \left[\Delta^{\mu'\nu'}(t; k, q_e) \right]^\dagger \gamma^0 \right] \quad (36)$$

Calculation of the trace and its convolution with $-g_{\mu\mu'}$ is straightforward but tedious even using the approximation (34). Some details are presented in Appendix B.

Integrating with respect to the final proton momentum \mathbf{p}_4 in the special frame, where $t = p_1 + p_2 - p_3$ has no spatial components (i.e. $t^0 = W$, $\mathbf{t} = 0$, where W is defined by $W^2 = (p_1 + p_2 - p_3)^2$), we come to

$$\frac{d\sigma_\Delta^{(1)}}{d\Omega} = \frac{1}{(4\pi)^2} \frac{1}{4M_p^2 \eta} \int \frac{\varepsilon_3 d\varepsilon_3}{\varepsilon_{3,\text{el}}} \frac{M_p}{W} \int \frac{\omega^2 d\Omega_\gamma}{(2\pi)^3 2\omega} \sum \left| \mathcal{M}_\Delta^{(1)} \right|^2, \quad (37)$$

where we use the limit $\varepsilon_1, \varepsilon_3 \gg m$:

$$\frac{W^2 - M_p^2}{2M_p \eta} = \varepsilon_{3,\text{el}} - \varepsilon_3, \quad \eta = 1 + \frac{2\varepsilon_1}{M_p} \sin^2 \frac{\theta}{2}, \quad \varepsilon_{3,\text{el}} = \frac{\varepsilon_1}{\eta}, \quad (38)$$

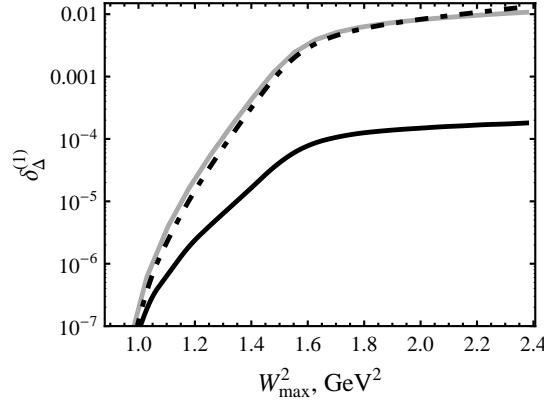


Figure 2. $\Delta(1232)$ contribution to real radiative corrections $\delta_{\Delta}^{(1)}$ for $E_{\text{beam}} = 1.594$ GeV, $Q^2 = 1.51$ GeV², i.e. for the point Run I, No. 1 at the VEPP-3 experiment [26]. Gray solid line represents the estimate (40); black dot-dashed line is numerical integration using the formula (37) with only W_{max}^2 restriction; black solid line is numerical integration with the proton emission angle cuts $\Delta\theta_p = \Delta\phi_p = 3^\circ$ corresponded to the VEPP-3 experimental point Run I, No. 1.

$\varepsilon_{3,\text{el}}$ is the final electron energy in the elastic scattering process; the photon energy in the special frame comes from the relation $W^2 = (p_4 + k)^2$:

$$\omega = \frac{W^2 - M_p^2}{2W}, \quad (39)$$

and $\int d\Omega_\gamma$ means the integration with respect to the photon directions in that special frame. The integration with respect to $d\varepsilon_3$ and $d\Omega_\gamma$ in (37) must be performed taking into account the particular experimental cuts. For the experiments with magnetic spectrometers (for example, the SLAC experiment [32]) we set the lower bound on the final electron energy $\varepsilon_{3,\text{el}} - \Delta E < \varepsilon_3 < \varepsilon_{3,\text{el}}$ and integrate over the total solid angle of the final photon directions. As for the VEPP-3 experiment [26], where the final electron and proton are detected in coincidence, there are a lower bound for the final electron energy $\varepsilon_{3,\text{el}} - \Delta E$ and the final proton angles cuts on the difference between the elastic and measured values ($\Delta\theta_p$ and $\Delta\phi_p$).

Using the approximation (34) and the formulas from the Appendix B we can find the contribution to real radiative corrections in the case of spectrometric experiments on cross section measurements:

$$\begin{aligned} \delta_{\Delta}^{(1)} &= \frac{d\sigma_{\Delta}^{(1)}/d\Omega}{d\sigma/d\Omega} \approx \frac{d\sigma'/d\Omega}{d\sigma/d\Omega} \frac{\Gamma_{\Delta \rightarrow \gamma p}}{\Gamma_{\Delta}} \\ &\times \frac{1}{\pi} \int_0^{2M_p \eta \Delta E} \frac{\Gamma_{\Delta} M_{\Delta}}{(x - M_{\Delta}^2 + M_p^2)^2 + \Gamma_{\Delta}^2 M_{\Delta}^2} \frac{x^3 dx}{(M_{\Delta}^2 - M_p^2)^3}, \end{aligned} \quad (40)$$

where $x = W^2 - M_p^2$. The integral in the right hand side is almost obvious: the square of Δ propagator yields the first multiplier; the powers of x (or ω) come from the photon phase space (ω^1) and from the matrix element, which starts with ω^1 in the soft photon limit, so its square is proportional to ω^2 ; the integrand is proportional to $\delta(x - M_{\Delta}^2 + M_p^2)$

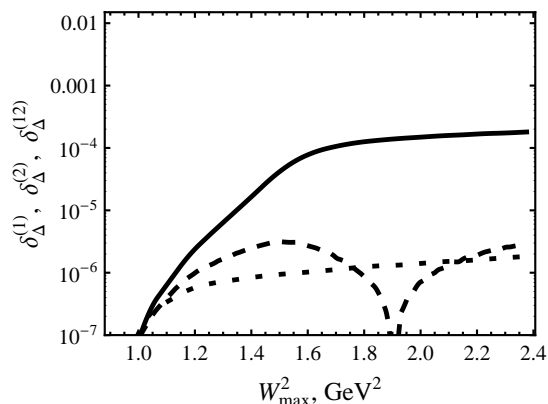


Figure 3. Various terms of $\Delta(1232)$ contribution to real radiative corrections: solid line shows $\delta_{\Delta}^{(1)}$, the contribution of $|\mathcal{M}_{\Delta}^{(1)}|^2$; dotted line is for $\delta_{\Delta}^{(2)}$, the contribution of $|\mathcal{M}_{\Delta}^{(2)}|^2$; dashed line is for absolute value $|\delta_{\Delta}^{(12)}|$ of the interference $2\text{Re}[\mathcal{M}_{\Delta}^{(1)}\mathcal{M}_{\Delta}^{(2)\dagger}]$ (the interference changes the sign from positive to negative at about $W_{\text{max}}^2 \simeq 1.9 \text{ GeV}^2$). Numerical integration is performed for $E_{\text{beam}} = 1.594 \text{ GeV}$, $Q^2 = 1.51 \text{ GeV}^2$ with the proton emission angle cuts $\Delta\theta_p = \Delta\phi_p = 3^\circ$ corresponded to the VEPP-3 experimental point Run I, No. 1.

in the limit $\Gamma_{\Delta} \rightarrow 0$, therefore the whole integral with the multiplier $1/\pi$ gives 1 in that limit.

So, indeed, there is the term proportional to the cross section ratio multiplied by the branching as we supposed in our rough estimate (11). But it is also multiplied by the factor, which appears to be very small for typical energy constraints in the experiments with magnetic spectrometers: $W_{\text{max}}^2 = M_p^2 + 2M_p\eta\Delta E < (M_p + m_{\pi})^2$, i.e. below the pion production threshold.

Numerical results for $\delta_{\Delta}^{(1)}$ are presented in Figure 2, where we show its dependence on the energy cut W_{max}^2 . In the first case we do not use any additional restrictions (elastic cross section measurements set-up). One can see that the approximate formula (40) is in quite good agreement with the full calculation of $|\mathcal{M}_{\Delta}^{(1)}|^2$. The typical value $W_{\text{max}}^2 = 1.12 \text{ GeV}^2$ (from the SLAC experiment [32]) leads to a strong suppression of $\Delta(1232)$ contribution. In the second case we perform integration with the final proton emission angle cuts which take place in the VEPP-3 experiment on measurements cross section ratio R . Here we see that for the conservative value $W_{\text{max}}^2 \approx 1.6 \text{ GeV}^2$ in Run I, No. 1 the smallness of the correction is primarily induced by the strict proton emission angle cuts. Here and in the following the full calculation of traces and numerical Monte-Carlo integration have been performed using FeynCalc [33, 34] and Wolfram Mathematica [35].

On the Figure 3 one can find that our assumption about $\mathcal{M}_{\Delta}^{(1)}$ dominance is in agreement with numerical results. We compare the contributions of $|\mathcal{M}_{\Delta}^{(1)}|^2$, $2\text{Re}[\mathcal{M}_{\Delta}^{(1)}\mathcal{M}_{\Delta}^{(2)\dagger}]$ and $|\mathcal{M}_{\Delta}^{(2)}|^2$. The second and the third contributions are lower than the first one, as we supposed. It should be noted that the regions $W^2 < M_{\Delta}^2$ and

$W^2 > M_\Delta^2$ work in opposite directions for the interference, so in particular situations the contribution can be suppressed and it will change the sign if W_{max}^2 is sufficiently greater than M_Δ^2 .

4.2. Delta(1232) contribution to real radiative corrections for the VEPP-3 experiment.

Here we investigate the C -odd interference of the proton bremsstrahlung with $\Delta(1232)$ in the intermediate state and the electron bremsstrahlung. Assuming the approximation (34) we decompose the electron bremsstrahlung into traditional “soft” and “hard” parts

$$i\mathcal{M}_e = i\mathcal{M}_e^{(s)} + i\mathcal{M}_e^{(h)}, \quad (41)$$

$$i\mathcal{M}_e^{(s)} = -\frac{iZe^3}{q_p^2} j_\nu(p_1, p_3) J_p^\nu(p_2, p_4) \left[\frac{p_3^\mu}{(p_3 k)} - \frac{p_1^\mu}{(p_1 k)} \right] \epsilon_\mu^*(k), \quad (42)$$

$$i\mathcal{M}_e^{(h)} = -\frac{iZe^3}{q_p^2} \bar{u}(p_3) \left(\frac{\gamma_\mu \hat{k} \gamma_\nu}{2(p_3 k)} + \frac{\gamma_\nu \hat{k} \gamma_\mu}{2(p_1 k)} \right) u(p_1) J_p^\nu(p_2, p_4) \epsilon^{*\mu}(k), \quad (43)$$

where $q_p = p_4 - p_2$ is the proton momentum transfer.

Our estimate for the interference is

$$2 \operatorname{Re} \left[\sum \bar{\mathcal{M}}_e^\dagger \mathcal{M}_\Delta \right] \approx 2 \operatorname{Re} \left[\sum \bar{\mathcal{M}}_e^{(s)\dagger} \mathcal{M}_\Delta^{(1)} \right]. \quad (44)$$

We can rewrite it as follows

$$\sum \bar{\mathcal{M}}_e^{(s)\dagger} \mathcal{M}_\Delta^{(1)} = \frac{Z^3 e^6}{q_e^2 q_p^2} \left[\frac{p_{3,\mu}}{(p_3 k)} - \frac{p_{1,\mu}}{(p_1 k)} \right] \frac{L_{\nu\nu'}(p_1, p_3) G^{\mu\nu\nu'}(t; k, q_e)}{t^2 - M_\Delta^2 + i\Gamma_\Delta M_\Delta}, \quad (45)$$

where

$$G^{\mu\nu\nu'}(t; k, q_e) = \frac{1}{2} \operatorname{Tr} \left[(\hat{p}_4 + M_p) \Delta^{\mu\nu}(t; k, q_e) (\hat{p}_2 + M_p) \Gamma_{\gamma p \rightarrow p}^{\nu'}(-q_p) \right]. \quad (46)$$

Some details can be found in the Appendix C. Here we present only the result within our approximation (34):

$$\begin{aligned} 2 \operatorname{Re} \left[\sum \bar{\mathcal{M}}_e^{(s)\dagger} \mathcal{M}_\Delta^{(1)} \right] &\approx \frac{Z^3 e^6}{(q^2)^2} \frac{2(W^2 - M_\Delta^2)}{(W^2 - M_\Delta^2)^2 + \Gamma_\Delta^2 M_\Delta^2} \frac{2G_1(0)(M_\Delta + M_p)}{3 M_\Delta^2} \\ &\times \frac{2M_p(KP)}{P^2} \left(G_E(q^2) G_M^*(q^2) + \frac{-q^2}{4M_p M_\Delta} G_M(q^2) G_C^*(q^2) \right) \\ &\times \left[\frac{p_{3,\mu}}{(p_3 k)} - \frac{p_{1,\mu}}{(p_1 k)} \right] (-g_{\lambda\lambda'}) \epsilon^{\lambda\tau\rho\mu} t_\tau k_\rho \epsilon^{\lambda'\tau'\sigma\nu} t_{\tau'} (q_e)_\sigma K_\nu, \end{aligned} \quad (47)$$

where $q_p^2 \approx q_e^2 \approx q^2$, $K = p_1 + p_3$, $P = p_2 + p_4$.

The contribution to real radiative corrections has the following form

$$\delta_\Delta^{(\text{int})} = \frac{d\sigma_\Delta^{(\text{int})}/d\Omega}{d\sigma/d\Omega}, \quad (48)$$

with

$$\frac{d\sigma_\Delta^{(\text{int})}}{d\Omega} = \frac{1}{(4\pi)^2} \frac{1}{4M_p^2 \eta} \int \frac{\varepsilon_3 d\varepsilon_3}{\varepsilon_{3,el}} \frac{M_p}{W} \int \frac{\omega^2 d\Omega_\gamma}{(2\pi)^3 2\omega} \sum 2 \operatorname{Re} [\mathcal{M}_e^\dagger \mathcal{M}_\Delta], \quad (49)$$

Table 1. $\Delta(1232)$ contribution to real radiative corrections in the VEPP-3 experiment [26].

	Run I, No. 1	Run I, No. 2	Run II, No. 1	Run II, No. 2
E_{beam} (GeV)	1.594	1.594	0.998	0.998
Q^2 (GeV ²)	1.51	0.298	0.976	0.830
$\Delta E/\varepsilon_{3,el}$	0.25	0.45	0.29	0.29
$\Delta\theta_p, \Delta\phi_p$	3.0°	5.0°	3.0°	3.0°
$\delta_{\Delta}^{(s,1)}, 10^{-5}$	0.64 ± 0.03	0.3 ± 0.1	2.96 ± 0.01	2.46 ± 0.01
$\delta_{\Delta}^{(h,1)}, 10^{-5}$	-0.75 ± 0.01	6.21 ± 0.05	-0.81 ± 0.01	-0.97 ± 0.01
$\delta_{\Delta}^{(s,2)}, 10^{-5}$	1.26 ± 0.02	-1.82 ± 0.03	1.20 ± 0.01	1.01 ± 0.01
$\delta_{\Delta}^{(h,2)}, 10^{-5}$	-0.88 ± 0.01	-1.58 ± 0.01	-0.49 ± 0.01	-0.55 ± 0.01
$\delta_{\Delta}^{(\text{int})}, 10^{-5}$	0.32 ± 0.02	3.2 ± 0.1	2.86 ± 0.01	1.95 ± 0.01

where the integration area is restricted by particular experimental cuts, as it was explained for the similar formula (37).

One can easily find that in the special frame ($t^0 = W$, $\mathbf{t} = 0$) the dependence of the interference in the soft photon approximation on the photon emission direction is determined by the factor

$$\begin{aligned}
2 \operatorname{Re} \left[\sum \overline{\mathcal{M}_e^{(s)\dagger}} \mathcal{M}_{\Delta}^{(1)} \right] &\propto \left[\frac{p_{3,\mu}}{(p_3 k)} - \frac{p_{1,\mu}}{(p_1 k)} \right] (-g_{\lambda\lambda'}) \epsilon^{\lambda\tau\rho\mu} t_{\tau} k_{\rho} \epsilon^{\lambda'\tau'\sigma\nu} t_{\tau'} (q_e)_{\sigma} K_{\nu} \\
&= W^2 \left(\left[\mathbf{k} \times \left(\frac{\mathbf{p}_3}{(p_3 k)} - \frac{\mathbf{p}_1}{(p_1 k)} \right) \right] \cdot [\mathbf{q}_e \times \mathbf{K}] \right), \tag{50}
\end{aligned}$$

where all vectors are considered in that special frame. Then the integration over the total solid angle yields to

$$\int d\Omega_{\gamma} \frac{\mathbf{k}}{(p_1 k)} \propto \mathbf{p}_1, \quad \int d\Omega_{\gamma} \frac{\mathbf{k}}{(p_3 k)} \propto \mathbf{p}_3, \tag{51}$$

so the first cross product in (50) and the whole interference within the approximations (34) and (44) yields zero for the experiments where all final photon directions in the special frame are possible (as it takes place in the experiments with magnetic spectrometers considered earlier).

But for the VEPP-3 experiment the integration over the total solid angle of the photon emission directions in the special frame is consistent with the proton angles cuts ($\Delta\theta_p$ and $\Delta\phi_p$) only within a certain range of ε_3 (not too much different from the $\varepsilon_{3,el}$). The actual area of integration with respect to the photon emission angles is complex. So the result can only be computed numerically. In the Table 1 we present the $\Delta(1232)$ contribution to real radiative corrections for the VEPP-3 experiment: the “soft” and “hard” part of the interference with $\mathcal{M}_{\Delta}^{(1)}$ and $\mathcal{M}_{\Delta}^{(2)}$; $\delta_{\Delta}^{(s,1)}$ comes from the contribution of $\mathcal{M}_e^{(s)\dagger} \mathcal{M}_{\Delta}^{(1)}$ and so on; the values are presented with the estimates of Monte Carlo integration errors; the full contribution $\delta_{\Delta}^{(\text{int})}$ was calculated independently on the “soft” and “hard” parts in order for an additional crosscheck, and within the error it is in agreement with the sum of the partial contributions. As one could expect, the results

show a strong dependence on the experimental conditions and cuts. We see that the soft photon approximation works better for the Run II conditions, and for the Run I it gives the answer only in the order of magnitude. Anyway the actual value of $\delta_{\Delta}^{(\text{int})} < 0.01\%$ ensures us that this contribution can not alter the results on the $e^{\pm}p$ cross sections ratio where the TPE effect is about 1%.

5. Conclusion

Here we considered the contribution of $\Delta(1232)$ resonance to real radiative corrections. It was shown that although the rough estimate gives the significant value, the actual results are typically suppressed by strict energy cuts or angular constraints. The effect is found to be negligible for past experiments to measure unpolarized elastic scattering cross section as well as for the recent experiment at the VEPP-3 storage ring to investigate the TPE effects.

6. Acknowledgments

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Appendix A. Current tensors

For the electron current tensor we have

$$\begin{aligned} L^{\nu\rho}(p_1, p_3) &= \sum_{\bar{}} j^{\nu}(p_1, p_3) j^{\dagger\rho}(p_1, p_3) \\ &= \frac{1}{2} \text{Tr} [(\hat{p}_3 + m) \gamma^{\nu} (\hat{p}_1 + m) \gamma^{\rho}] \\ &= q^2 g^{\nu\rho} - q^{\nu} q^{\rho} + K^{\nu} K^{\rho}, \end{aligned} \quad (\text{A.1})$$

where $K = p_1 + p_3$, $q = p_1 - p_3$.

The proton current tensor is

$$\begin{aligned} T_p^{\nu\rho}(p_2, p_4) &= \sum_{\bar{}} J_p^{\nu}(p_2, p_4) J_p^{\dagger\rho}(p_2, p_4) \\ &= \frac{1}{2} \text{Tr} [(\hat{p}_4 + M_p) \Gamma_{\gamma p \rightarrow p}^{\nu}(q) (\hat{p}_2 + M_p) \Gamma_{\gamma p \rightarrow p}^{\rho}(-q)] \\ &= 4M_p^2 \left[\frac{-q^2}{4M_p^2} G_M^2(q^2) \left(-g^{\nu\rho} + \frac{q^{\nu} q^{\rho}}{q^2} + \frac{P^{\nu} P^{\rho}}{P^2} \right) + G_E^2(q^2) \frac{P^{\nu} P^{\rho}}{P^2} \right], \end{aligned} \quad (\text{A.2})$$

where $P = p_2 + p_4$, $q = p_4 - p_2$.

And the transition current tensor is

$$\begin{aligned} T_{p \rightarrow \Delta}^{\nu\rho}(p_2, p'_4) &= \sum_{\bar{}} J_{p \rightarrow \Delta}^{\nu}(p_2, p'_4) J_{p \rightarrow \Delta}^{\dagger\rho}(p_2, p'_4) \\ &= \frac{1}{2} \text{Tr} \left[(\hat{p}'_4 + M_{\Delta}) \mathcal{P}_{\alpha\beta}(p'_4) \Gamma_{\gamma p \rightarrow \Delta}^{\nu\beta}(p'_4, q') (\hat{p}_2 + M_p) \Gamma_{\Delta \rightarrow \gamma p}^{\rho\alpha}(p'_4, q') \right] \\ &= \frac{(M_{\Delta} + M_p)^2}{4M_p^2} ((M_{\Delta} - M_p)^2 - q'^2) \end{aligned}$$

$$\begin{aligned} & \times \left[\left(G_M^{*2}(q'^2) + 3G_E^{*2}(q'^2) \right) \left(-g^{\mu\nu} + \frac{q'^\mu q'^\nu}{q'^2} + \frac{\tilde{P}^\mu \tilde{P}^\nu}{\tilde{P}^2} \right) \right. \\ & \left. + \frac{-q'^2}{M_\Delta^2} G_C^{*2}(q'^2) \frac{\tilde{P}^\mu \tilde{P}^\nu}{\tilde{P}^2} \right], \end{aligned} \quad (\text{A.3})$$

where $q' = p'_4 - p_2$,

$$\tilde{P}^\mu = P' - \frac{(P' \cdot q')}{q'^2} q'^\mu, \quad P' = p_2 + p'_4, \quad (\text{A.4})$$

and we use the sum over Δ -particle polarization states

$$\sum U_\alpha(t) \bar{U}_\beta(t) = (\hat{t} + M_\Delta) \mathcal{P}_{\alpha\beta}(t), \quad (\text{A.5})$$

with $\mathcal{P}_{\alpha\beta}(t)$ defined in (33).

For the elastic scattering process $ep \rightarrow ep$:

$$L_{\nu\rho}(p_1, p_3) T_p^{\nu\rho}(p_2, p_4) = ((KP)^2 + q^2 P^2) \frac{\tau G_M^2(q^2) + \epsilon G_E^2(q^2)}{\epsilon(1 + \tau)}, \quad (\text{A.6})$$

where

$$\tau = \frac{-q^2}{4M_p^2}, \quad \epsilon = \frac{(KP)^2 + q^2 P^2}{(KP)^2 - K^2 P^2 - 2q^2 P^2}, \quad (\text{A.7})$$

and we used

$$(Kq) = 0, \quad (Pq) = 0. \quad (\text{A.8})$$

If we consider the case with ultrarelativistic electrons ($\varepsilon_1, \varepsilon_3 \gg m$, $q^2 \gg m^2$), we will have

$$L_{\nu\rho}(p_1, p_3) T_p^{\nu\rho}(p_2, p_4) = 4M_p^2 \left(4\varepsilon_1 \varepsilon_3 \cos^2 \frac{\theta}{2} \right) \frac{\tau G_M^2(q^2) + \epsilon G_E^2(q^2)}{\epsilon(1 + \tau)} \quad (\text{A.9})$$

with τ and ϵ defined in (24).

As for the process $ep \rightarrow e\Delta$:

$$\begin{aligned} L_{\nu\rho}(p_1, p'_3) T_{p \rightarrow \Delta}^{\nu\rho}(p_2, p'_4) &= \left((K'\tilde{P})^2 + q'^2 \tilde{P}^2 \right) \frac{(M_\Delta + M_p)^2}{4M_p^2} \\ &\times \frac{\tau' \left(G_M^{*2}(q'^2) + 3G_E^{*2}(q'^2) + \epsilon' \frac{-q'^2}{M_\Delta^2} G_C^{*2}(q'^2) \right)}{\epsilon'(1 + \tau')}, \end{aligned} \quad (\text{A.10})$$

where

$$\tau' = \frac{-q'^2}{(M_p + M_\Delta)^2}, \quad \epsilon' = \frac{(K'\tilde{P})^2 + q'^2 \tilde{P}^2}{(K'\tilde{P})^2 - K'^2 \tilde{P}^2 - 2q'^2 \tilde{P}^2}, \quad (\text{A.11})$$

and we used

$$(K'q') = 0, \quad (\tilde{P}q') = 0, \quad (\text{A.12})$$

$$\tilde{P}^2 = \frac{P'^2 q'^2 - (P'q')^2}{q'^2} = \frac{((M_\Delta - M_p)^2 - q'^2)((M_\Delta + M_p)^2 - q'^2)}{-q'^2}, \quad (\text{A.13})$$

where $K' = p_1 - p'_3$.

Again in the case of ultrarelativistic electrons ($\varepsilon_1, \varepsilon'_3 \gg m$, $q'^2 \gg m^2$) we have

$$L_{\nu\rho}(p_1, p'_3) T_{p \rightarrow \Delta}^{\nu\rho}(p_2, p'_4) = 4M_p^2 \left(4\varepsilon_1 \varepsilon'_3 \cos^2 \frac{\theta}{2} \right) \frac{(M_\Delta + M_p)^2}{4M_p^2} \times \frac{\tau' \left(G_M^{*2}(q'^2) + 3G_E^{*2}(q'^2) + \epsilon' \frac{-q'^2}{M_\Delta^2} G_C^{*2}(q'^2) \right)}{\epsilon'(1 + \tau')}, \quad (\text{A.14})$$

with τ' and ϵ' defined in (26).

Appendix B. Approximation for $|\mathcal{M}_\Delta^{(1)}|^2$

To calculate the matrix element $\mathcal{M}_\Delta^{(1)}$ it is useful to consider it in the special frame, where the 4-vector t has no spatial components $t = p_1 + p_2 - p_3$, $t = \{W, 0\}$. We have in this special frame

$$\begin{aligned} q_e &= \{q_e^0, \mathbf{q}_e\}, & p_2 &= \{\mathcal{E}_2, -\mathbf{q}_e\}, \\ k &= \{\omega, \mathbf{k}\}, & p_4 &= \{\mathcal{E}_4, -\mathbf{k}\}, \end{aligned} \quad (\text{B.1})$$

where

$$q_e^0 = \frac{W^2 - M_p^2 + q_e^2}{2W}, \quad \mathcal{E}_2 = \frac{W^2 + M_p^2 - q_e^2}{2W}, \quad (\text{B.2})$$

$$\omega = \frac{W^2 - M_p^2}{2W}, \quad \mathcal{E}_4 = \frac{W^2 + M_p^2}{2W},$$

$$|\mathbf{q}_e| = \frac{\sqrt{(W - M_p)^2 - q_e^2} \sqrt{(W + M_p)^2 - q_e^2}}{2W}. \quad (\text{B.3})$$

The soft photon approximation means

$$W \rightarrow M_p, \quad \mathcal{E}_4 \rightarrow M_p. \quad (\text{B.4})$$

One can easily check that in the special frame the numerator of the Δ propagator (33) is equal to zero for time-like indexes:

$$\mathcal{P}^{0\beta}(t) = \mathcal{P}^{\alpha 0}(t) = 0. \quad (\text{B.5})$$

For spatial indexes $a, b = 1, 2, 3$ we have (here and after we use Latin letters for spatial components of 4-vectors and tensors):

$$(\hat{t} + M_\Delta) \mathcal{P}^{ab}(t) \approx \frac{2M_\Delta}{3} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes (2\delta^{ab} - i\epsilon^{abc} \boldsymbol{\sigma}^c), \quad (\text{B.6})$$

where we have dropped the terms proportional to $W - M_\Delta$. Here we use the standard representation of the Dirac γ -matrices, the Pauli σ -matrices, and the spatial Levi-Civita tensor ϵ^{abc} .

Let us consider the vertex with the real photon emission in the special frame:

$$\Gamma_{\Delta \rightarrow \gamma p}^{0a}(t, k) \approx -\sqrt{\frac{2}{3}} \frac{W}{2M_\Delta^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes [G_1(0) i\epsilon^{acd} \mathbf{k}^c \boldsymbol{\sigma}^d - G_2(0) \mathbf{k}^a], \quad (\text{B.7})$$

and

$$\Gamma_{\Delta \rightarrow \gamma p}^{ma}(t, k) \approx -\sqrt{\frac{2}{3}} \frac{W}{2M_\Delta^2} \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes [G_1(0) i\epsilon^{amc} \omega \boldsymbol{\sigma}^c - G_2(0) \delta^{ma} \omega] \right. \\ \left. - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes [G_1(0) (\delta^{ma}(\boldsymbol{\sigma} \mathbf{k}) - \boldsymbol{\sigma}^m \mathbf{k}^a)] \right\}, \quad (\text{B.8})$$

where we dropped the term with $G_3(0)$ because it is proportional to ω^2 .

The vertex with the virtual photon absorption have the following form

$$\Gamma_{\gamma p \rightarrow \Delta}^{0b}(t, q_e) = -\sqrt{\frac{2}{3}} \frac{W}{2M_\Delta^2} \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes [G_1(q_e^2) i\epsilon^{bgf} \mathbf{q}_e^g \boldsymbol{\sigma}^f + G_2(q_e^2) \mathbf{q}_e^b] \right. \\ \left. - \frac{G_3(q_e^2)}{M_\Delta} \left[-\mathbf{q}_e^2 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \boldsymbol{\sigma}^b + q_e^0 \mathbf{q}_e^b \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right] \right\} \quad (\text{B.9})$$

and

$$\Gamma_{\gamma p \rightarrow \Delta}^{nb}(t, q_e) = -\sqrt{\frac{2}{3}} \frac{W}{2M_\Delta^2} \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes [G_1(q_e^2) i\epsilon^{bne} q_e^0 \boldsymbol{\sigma}^e + G_2(q_e^2) \delta^{nb} q_e^0] \right. \\ \left. - \frac{G_3(q_e^2)}{M_\Delta} \left[(q_e^2 \delta^{nb} + \mathbf{q}_e^n \mathbf{q}_e^b) \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - \mathbf{q}_e^n q_e^0 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \boldsymbol{\sigma}^b \right] \right. \\ \left. - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes [G_1(0) (\delta^{nb}(\boldsymbol{\sigma} \mathbf{q}_e) - \boldsymbol{\sigma}^n \mathbf{q}_e^b)] \right\}. \quad (\text{B.10})$$

In the soft photon limit the final proton bispinor has only top components

$$U(p_4) \approx \left\{ \sqrt{\mathcal{E}_4 + M_p} \varphi_4, 0 \right\}, \quad (\text{B.11})$$

while the bottom components contain to $\sqrt{\mathcal{E}_4 - M_p} \approx \sqrt{\omega^2/2M_p}$.

Taking into account the formulas (B.5)–(B.11) we can obtain the approximation

$$\Delta^{m0} \approx -\frac{G_1(0)}{9} \frac{W^2}{M_\Delta^3} \frac{\tilde{G}_C(q_e^2)}{2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes (2i\epsilon^{mlp} \mathbf{k}^l - \mathbf{k}^p \boldsymbol{\sigma}^m + \delta^{mp}(\boldsymbol{\sigma} \mathbf{k})) \mathbf{q}_e^p \quad (\text{B.12})$$

and

$$\Delta^{mn} \approx \frac{G_1(0)}{9} \frac{W^2}{M_\Delta^3} \left\{ -\frac{\tilde{G}_C(q_e^2)}{2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes (2i\epsilon^{mln} \mathbf{k}^l - \mathbf{k}^n \boldsymbol{\sigma}^m + \delta^{mn}(\boldsymbol{\sigma} \mathbf{k})) q_e^0 \right. \\ \left. + \frac{G_3(q_e^2)}{M_\Delta} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes (2i\epsilon^{mlr} \mathbf{k}^l - \mathbf{k}^r \boldsymbol{\sigma}^m + \delta^{mr}(\boldsymbol{\sigma} \mathbf{k})) (q_e^2 \delta^{nr} - \mathbf{q}_e^n \mathbf{q}_e^r) \right. \\ \left. + G_1(q_e^2) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes [\epsilon^{xml} \boldsymbol{\sigma}^l \epsilon^{xar} \mathbf{k}^r (2\delta^{ab} - i\epsilon^{abc} \boldsymbol{\sigma}^c) \epsilon^{x'nl'} \boldsymbol{\sigma}^{l'} \epsilon^{x'br'} \mathbf{q}_e^{r'}] \right\}, \quad (\text{B.13})$$

where we introduced

$$\frac{\tilde{G}_C(q_e^2)}{2} = -(G_1(q_e^2) - G_2(q_e^2)) + G_3(q_e^2) \frac{q_e^0}{M_\Delta}. \quad (\text{B.14})$$

Strictly speaking our approximation (34) implies $W = M_p$, q_e to be equal q (the momentum transfer in the elastic scattering), no difference between M_Δ and M_p and some other relations. But since it is possible to identify the presented terms in the full matrix element and trace calculation results we do not perform all of these transformations here and in the following section.

Taking into account that the integration with respect to all real photon directions leads to

$$\mathbf{k}^i \mathbf{k}^j \rightarrow \frac{\omega^2}{3} \delta^{ij}, \quad (\text{B.15})$$

we will write down the averaged value of $\bar{H}^{\nu\nu'} = \int H^{\nu\nu'} d\Omega_\gamma / 4\pi$:

$$\begin{aligned} \bar{H}^{00} &= \frac{G_1^2(0)}{9^2} \frac{W^4 \omega^2}{M_\Delta^6} (\mathcal{E}_4 + M_p)(\mathcal{E}_2 - M_p) \tilde{G}_C^2(q_e^2) \mathbf{q}_e^2, \\ \bar{H}^{n0} &= \frac{G_1^2(0)}{9^2} \frac{W^4 \omega^2}{M_\Delta^6} (\mathcal{E}_4 + M_p)(\mathcal{E}_2 - M_p) \tilde{G}_C^2(q_e^2) \mathbf{q}_e^n q_e^0, \\ \bar{H}^{nn'} &= \frac{G_1^2(0)}{9^2} \frac{W^4 \omega^2}{M_\Delta^6} (\mathcal{E}_4 + M_p)(\mathcal{E}_2 - M_p) \left\{ \tilde{G}_C^2(q_e^2) \frac{\mathbf{q}_e^n \mathbf{q}_e^{n'}}{q_e^2} q_0^2 \right. \\ &\quad \left. + M_\Delta^2 \left(\tilde{G}_M^2(q_e^2) + 3\tilde{G}_E(q_e^2) \right) \left(\delta^{nn'} - \frac{\mathbf{q}_e^n \mathbf{q}_e^{n'}}{q_e^2} \right) \right\}, \end{aligned} \quad (\text{B.16})$$

where it was useful to introduce $\tilde{G}_{M,E}$ in addition to \tilde{G}_C (B.14):

$$\begin{aligned} \frac{\tilde{G}_M(q_e^2) - \tilde{G}_E(q_e^2)}{2} &= \frac{\mathcal{E}_2 + M_p}{M_\Delta} G_1(q_e^2), \\ \tilde{G}_E(q_e^2) &= -\frac{q_e^0}{M_\Delta} (G_1(q_e^2) - G_2(q_e^2)) + G_3(q_e^2) \frac{q_e^2}{M_\Delta^2}. \end{aligned} \quad (\text{B.17})$$

These quantities can be reduced to $G_{M,E,C}$ for $W = M_\Delta$

$$\tilde{G}_{M,E,C}(q_e^2)|_{W=M_\Delta} = \frac{3(M_\Delta + M_p)}{M_p} G_{M,E,C}^*(q_e^2), \quad (\text{B.18})$$

The tensor $\bar{H}^{\nu\nu'}$ at the point $W = M_\Delta$ can be rewritten in terms of the transition current tensor $T_{p \rightarrow \Delta}$ and the partial width $\Gamma_{\Delta \rightarrow p\gamma}$:

$$\bar{H}^{\nu\nu'}|_{W=M_\Delta} \approx \frac{64\pi \Gamma_{\Delta \rightarrow p\gamma}}{Z^2 e^2} \frac{M_\Delta^5 \omega^2}{(M_\Delta^2 - M_p^2)^3} T_{p \rightarrow \Delta}^{\nu\nu'}(p_2, t)|_{W=M_\Delta}, \quad (\text{B.19})$$

where

$$\begin{aligned} \Gamma_{\Delta \rightarrow p\gamma} &= \frac{\sum |\mathcal{M}_{\Delta \rightarrow p\gamma}|^2}{16\pi} \frac{M_\Delta^2 - M_p^2}{M_\Delta^3} = \frac{-T_{p \rightarrow \Delta}^{\nu\nu}|_{W=M_\Delta, q^2=0}}{2} \frac{M_\Delta^2 - M_p^2}{16\pi M_\Delta^3} \\ &= \frac{Z^2 e^2 (M_\Delta^2 - M_p^2)^3}{64\pi M_p^2 M_\Delta^3} [G_M^{*2}(0) + 3G_E^{*2}(0)] \\ &\approx \frac{Z^2 e^2 (M_\Delta^2 - M_p^2)^3}{144\pi M_\Delta^3} G_1^2(0). \end{aligned} \quad (\text{B.20})$$

Finally, we have the following expression for the differential cross section (37):

$$\begin{aligned} \frac{d\sigma_{\Delta}^{(1)}}{d\Omega} &\approx \frac{d\sigma'}{d\Omega} \frac{\Gamma_{\Delta \rightarrow \gamma p}}{\Gamma_{\Delta}} \\ &\times \frac{1}{\pi} \int_0^{2M_p \eta \Delta E} \left[\frac{\Gamma_{\Delta} M_{\Delta}}{(x + M_p^2 - M_{\Delta}^2)^2 + \Gamma_{\Delta}^2 M_{\Delta}^2} \right] \frac{x^3 dx}{(M_{\Delta}^2 - M_p^2)^3}, \end{aligned} \quad (\text{B.21})$$

where we used our approximation (34):

$$\begin{aligned} x &= W^2 - M_p^2, & dx &\approx 2M_{\Delta} dW, \\ W dW &= -M_p \eta d\varepsilon_3, & \omega &\approx \frac{x}{2M_{\Delta}}, \end{aligned} \quad (\text{B.22})$$

and

$$\frac{d\sigma'}{d\Omega} = \frac{1}{(4\pi)^2} \frac{1}{4M_p^2 \eta} \frac{\varepsilon_3}{\eta \varepsilon_{3,el}} \frac{Z^2 e^2}{(q_e^2)^2} L_{\nu\nu'}(p_1, p_3) T_{p \rightarrow \Delta}^{\nu\nu'}(p_2, t) \Big|_{W=M_{\Delta}}. \quad (\text{B.23})$$

Appendix C. Approximation for the interference $\mathcal{M}_e^{(s)\dagger} \mathcal{M}_{\Delta}^{(1)}$

Here we consider the tensor, which appears in the interference:

$$G^{\mu\nu\nu'}(t; k, q_e) = \frac{1}{2} \text{Tr} \left[(\hat{p}_4 + M_p) \Delta^{\mu\nu}(t; k, q_e) (\hat{p}_2 + M_p) \Gamma_{\gamma p \rightarrow p}^{\nu'}(-q_p) \right]. \quad (\text{C.1})$$

Here for real protons one can make the substitution

$$\Gamma_{\gamma p \rightarrow p}^{\nu}(-q_p) = 2M_p (G_E(q_p^2) - G_M(q_p^2)) \frac{P^{\nu}}{P^2} + G_M(q_p^2) \gamma^{\nu}, \quad (\text{C.2})$$

where $P = p_2 + p_4$. Therefore it is possible to decompose the tensor G :

$$G^{\mu\nu\nu'}(t; k, q_e) = \frac{2M_p G_E(q_p^2)}{P^2} P^{\nu'} G_1^{\mu\nu} + G_M(q_p^2) G_2^{\mu\nu\nu'}. \quad (\text{C.3})$$

A straightforward calculation with approximate values of $\Delta^{\mu\nu}$ from (B.12) and (B.13) leads to the following values in the special frame:

$$\begin{aligned} G_1^{m0} &= 0, \\ G_1^{mn} &= \frac{G_1(0)}{9} \frac{W^2}{M_{\Delta}^3} (\mathcal{E}_4 + M_p) ((\mathbf{k} \mathbf{q}_e)^{\delta mn} - \mathbf{q}_e^m \mathbf{k}^n) M_{\Delta} \tilde{G}_M(q_e^2). \end{aligned} \quad (\text{C.4})$$

It is worth to note that these tensors appear in convolution with the symmetric tensor $L_{\nu\nu'}$. The symmetrized values for the second tensor ($\tilde{G}_2^{m\nu\nu'} = (G_2^{m\nu\nu'} + G_2^{m\nu'\nu})/2$):

$$\begin{aligned} \tilde{G}_2^{m00} &= 0, \\ \tilde{G}_2^{m0n} &= \frac{G_1(0)}{9} \frac{W^2}{M_{\Delta}^3} (\mathcal{E}_4 + M_p) \frac{((\mathbf{k} \mathbf{q}_e)^{\delta mn} - \mathbf{q}_e^m \mathbf{k}^n)}{2} \\ &\times \left[\frac{(\mathcal{E}_2 - M_p)}{2} \tilde{G}_C(q_e^2) + \left(1 - \frac{2M_p(\mathcal{E}_2 + \mathcal{E}_4)}{P^2} \right) M_{\Delta} \tilde{G}_M(q_e^2) \right], \\ \tilde{G}_2^{mnn'} &= \frac{G_1(0)}{9} \frac{W^2}{M_{\Delta}^3} (\mathcal{E}_4 + M_p) \\ &\times \left(\mathbf{k}^{n'} \mathbf{q}_e^m \mathbf{q}_e^n - \mathbf{q}_e^n (\mathbf{k} \mathbf{q}_e)^{\delta mn'} + \mathbf{k}^n \mathbf{q}_e^m \mathbf{q}_e^{n'} - \mathbf{q}_e^{n'} (\mathbf{k} \mathbf{q}_e)^{\delta mn} \right) \\ &\times \left[G_1(q_e^2) - G_3(q_e^2) \frac{(\mathcal{E}_2 - M_p)}{2M_{\Delta}} - \frac{M_p M_{\Delta}}{P^2} \tilde{G}_M(q_e^2) \right], \end{aligned} \quad (\text{C.5})$$

where we used $\mathbf{P} \approx -\mathbf{q}_e$ in the soft photon limit.

Finally, using (34) we can find the approximate result

$$\begin{aligned}
G^{\mu\nu'}(t; k, q_e) \approx & \frac{2G_1(0)}{3} \frac{(M_\Delta + M_p)}{M_\Delta^3} \frac{2M_p}{P^2} \\
& \times \left(M_\Delta G_E(q_p^2) G_M^*(q_e^2) + \frac{-q_e^2}{4M_p} G_M(q_p^2) G_C^*(q_e^2) \right) \\
& \times P^{\nu'}(-g_{\lambda\lambda'}) \epsilon^{\lambda\tau\rho\mu} t_\tau k_\rho \epsilon^{\lambda'\tau'\sigma\nu} t_{\tau'}(q_e)_\sigma .
\end{aligned} \tag{C.6}$$

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